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INVERSE PROBLEMS OF ELASTOPLASTIC FILTRATION OF LIQUID IN A POROUS MEDIUM

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Inverse coefficient problems for equations of elastoplastic filtration of fluids in a porous medium in the regime of reduction and recovery of pressure have been solved by applying the methods of identification of parameters and deterministic moments. Examples of determining the piezoconductivities of a bed in the regimes of reduction and recovery of pressure in a well are given, where the values of the solution of the corresponding direct problem determined at the given points of the bed are used as the initial information.

Problems of elastoplastic filtration of fluids are of great interest in connection with the exploitation of oil and gas fields at great depths. In deep-lying oil and gas deposits, especially at anomalously high bed pressures, the pressure in a bed is reduced appreciably during exploitation, which leads to the appearance of high effective stresses on the bed. This in turn causes considerable deformations of the skeleton of bed rocks. One of the characteristic features of the occurrence of deformation of a bed at large depths is the disturbance of the deformation elasticity accompanied by the appearance of plastic (irreversible) changes in the bed characteristics. For the first time, the irreversibility of the change in the penetrability of oil beds has been noted in [1, 2], although in petroleum practice such reductions in the collector properties of beds during the time of their exploitation (i.e., a decrease in bed pressure) have long been known. Model schematization of an elastoplastic regime of filtration has been suggested for the first time in [3, 4], and it formed the basis for the formation of notions of elastoplastic filtration as a whole, as well as of estimation of its characteristics. We note that in [3, 4], in both the regime of reduction and recovery of bed pressure, basic relations of the dynamics of porosity, permeability of a bed, and of fluid viscosity depending on the current pressure were taken as linear. With allowance for the nonlinearity of the deformation of a bed [5], the equations of the elastoplastic regime of filtration are given in [6]. At large depths oil pools adapted to slightly cemented terrigenous traps, especially with an anomalously high bed pressure, lose their stability and integrity with increase in effective stresses. This leads to destruction of the skeleton of the bed rocks and to sweeping out of the solid particles formed onto the surface together with the extracted oil. This can cause great technological troubles in the process of both oil extraction and opening up of the field. Models of elastoplastic filtration of fluid in unstable collectors are suggested in [7, 8]. An analysis of the results of implementation of the model shows that in unstable collectors the factors of irreversibility of deformation and instability of a bed exert reciprocal back actions on the collector properties of the bed.

The inverse coefficient problems of filtration in an elastic regime for beds with porous and fractured-porous collectors were considered to be of particular advantage and were used for developing engineering procedures for determining the bed parameters. However, in an elastoplastic regime the inverse problems virtually have not been studied; the evaluation of the plastic properties of rocks was made on the basis of hysteresis changes in the collector properties of a bed in the regime of reduction and recovery of pressure. In the present work, for the model suggested in [3, 4] inverse coefficient problems have been solved, and the piezoconductivities of a bed in the regimes of reduction and recovery of pressure have been estimated. In order to check the stability of the inverse problems, the initial information was obtained by solving the corresponding direct problems with prescribed coefficients of the equations. To solve the problems posed we use the methods of identification [9, 10] and of deterministic moments [11]. It should be noted that in [10] a wide class of inverse problems described by partial differential parabolic-type equations was

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investigated. The equations of elastoplastic filtration of a fluid in a porous medium in the one-dimensional case [3] have the form

$$\downarrow \frac{\partial p}{\partial t} = a_1 \frac{\partial^2 p}{\partial x^2}\,, \quad \uparrow \frac{\partial p}{\partial t} = a_2 \frac{\partial^2 p}{\partial x^2}\,, \quad a_1, a_2 = \text{const} > 0\,. \tag{1}$$

First, we consider the problem of determining a_1 by the identification method. Let x_1, x_2, ..., x_n be the characteristic points of the bed at which measurements of reductions in the bed pressure, which are well-known functions of time $z_j(t)$, $1 \le j \le n$, and $0 \le t \le T$, are made.

We will seek a_1 from the condition of the functional minimum:

$$J(a_1) = \sum_{j=1}^{n} \int_0^T \left[p(x_j, t) - z_j(t) \right]^2 dt\,. \tag{2}$$

The condition of stationarity of functional (2) will have the form

$$\frac{dJ(a_1)}{da_1} = 2 \sum_{j=1}^{n} \int_0^T \left[p(x_j, t) - z_j(t) \right] w(x_j, t)\, dt = 0\,, \tag{3}$$

where $w = \partial p / \partial a_1$. We will expound into a series the function p in the vicinity of $\overset{m}{a}_1$ to within second-order terms:

$$\overset{m+1}{p}(x, t) \approx \overset{m}{p}(x, t) + \left(\overset{m+1}{a}_1 - \overset{m}{a}_1 \right) \overset{m}{w}(x, t)\,. \tag{4}$$

Having substituted expansion (4) into (3) instead of p, we obtain a linearlized relation for the parameter $\overset{m+1}{a}_1$:

$$2 \sum_j \int_0^T \left[\overset{m}{p}(x_j, t) + \left(\overset{m+1}{a}_1 - \overset{m}{a}_1 \right) \overset{m}{w}(x_j, t) - z_j(t) \right] \overset{m}{w}(x_j, t)\, dt = 0\,,$$

whence the following approximation of $\overset{m+1}{a}_1$ can easily be calculated if the functions $\overset{m}{p}(x, t)$ and $\overset{m}{w}(x, t)$ are known:

$$\overset{m+1}{a}_1 = \frac{\displaystyle\sum_j \int_0^T \left[\overset{m}{a}_1 \overset{m}{w}(x_j, t) - \overset{m}{p}(x_j, t) + z_j(t) \right] \overset{m}{w}(x_j, t)\, dt}{\displaystyle\sum_j \int_0^T \overset{m}{w}{}^2(x_j, t)\, dt}\,. \tag{5}$$

To obtain an equation for the function $\overset{m}{w}(x, t)$, we linearize the first equation of (1) with respect to the solution on the lower iteration layer:

$$\frac{\partial \overset{m+1}{p}}{\partial t} = \overset{m}{a}_1 \frac{\partial^2 \overset{m}{p}}{\partial x^2} + \frac{\partial^2 \overset{m}{p}}{\partial x^2} \delta a_1 + \overset{m}{a}_1 \left(\frac{\partial \overset{m+1}{p}}{\partial x^2} - \frac{\partial^2 \overset{m}{p}}{\partial x^2} \right), \quad \delta a_1 = \overset{m+1}{a}_1 - \overset{m}{a}_1\,. \tag{6}$$

Substituting expansion (4) into Eq. (6) and equating to zero the coefficients at δa_1, we find the following system of equations:

$$\frac{\partial \overset{m}{p}}{\partial t} = \overset{m}{a}_1 \frac{\partial^2 \overset{m}{p}}{\partial x^2}\,, \quad \frac{\partial \overset{m}{w}}{\partial t} = \overset{m}{a}_1 \frac{\partial^2 \overset{m}{w}}{\partial x^2} + \frac{\partial^2 \overset{m}{p}}{\partial x^2}\,, \tag{7}$$

which, at a given value of $\overset{m}{a}_1$, can be solved successively by one of the well-known numerical methods. The boundary and initial conditions for the function w result from the corresponding conditions for the function p by differentiating them with respect to the parameter a_1.

We will consider the numerical implementation of the above method using as an example the determination of the parameter a_1 in the first equation of (1) in the final bed [0, L] with the initial and boundary conditions

$$p\,(x, 0) = p_0\,, \quad p\,(0, t) = p_{\mathrm{w}}\,, \quad p\,(L, t) = p_0\,, \quad p_0, p_{\mathrm{w}} = \mathrm{const}\,. \tag{8}$$

First, we numerically solve the first equation with conditions (8) at the given value of a_1 and find the solution at the points x_j, $j = \overline{1, n}$. Then as the "data of measurements" we use $z(x_j, t_k) = p(x_j, t_k)$, where t_k is the discrete time for which the solution $p(x, t)$ has been found. This time is selected from the grid time layer used subsequently for a difference solution of the problem. The values of $z(x_j, t_k)$ were calculated at five points x_j = 5, 10, 15, 20, and 25 m for different values of t_k. The first equation of (7) is also solved with conditions (8) and the second one — with the following conditions:

$$w\,(x, 0) = 0\,, \quad w\,(0, t) = 0\,, \quad w\,(L, t) = 0\,. \tag{9}$$

The numerical algorithm of finding a_1 can be constructed as follows: a) we prescribe a certain initial approximation $\overset{0}{a}_1$ (we assume that m = 0); b) we solve system (7) with conditions (8) and (9) determining the functions $\overset{m}{p}$ and $\overset{m}{w}$; c) then (2) and (5) are calculated; d) assuming that $a_1 = \overset{m+1}{a}_1$, we repeat stages b) and c) until the needed accuracy is reached.

As the criterion of the end of the iteration process one of the following inequalities can be used:

$$\left|\overset{m+1}{p} - \overset{m}{p}\right| < \varepsilon_1\,, \quad \left|\overset{m+1}{a}_1 - \overset{m}{a}_1\right| < \varepsilon_2\,, \quad \left|J\left(\overset{m+1}{a}_1\right) - J\left(\overset{m}{a}_1\right)\right| < \varepsilon_3$$

or they taken together. The second equation of (1) in the regime of pressure recovery is solved under the following conditions:

$$p\,(x, 0) = p_1\,(x)\,, \quad \left.\frac{\partial p}{\partial x}\right|_{x=0} = 0\,, \quad p\,(L, t) = p_0\,. \tag{10}$$

We solve the system of equations (7) by the difference method [12]. In the region $\mathrm{D}\left\{0 \le x \le L, 0 \le t \le T\right\}$ we introduce the grid $\omega_{h\tau} = \left\{(x_i, t_k), i = \overline{0, I}, k = \overline{0, K}, x_i = ih, t_k = k\tau, h = L/I, \tau = T/K\right\}$. We approximate the system of equations (7) on the grid $\omega_{h\tau}$ by an implicit finite-difference scheme to within $O(\tau + h^2)$ (not indicating the iteration number over p):

$$\frac{\overset{k+1}{p_i} - \overset{k}{p_i}}{\tau} = \overset{m}{a}_1 \frac{\overset{k+1}{p}_{i+1} - 2\overset{k+1}{p_i} + \overset{k+1}{p}_{i-1}}{h^2}\,, \quad \frac{\overset{k+1}{w_i} - \overset{k}{w_i}}{\tau} = \overset{m}{a}_1 \frac{\overset{k+1}{w}_{i+1} - 2\overset{k+1}{w_i} + \overset{k+1}{w}_{i-1}}{h^2} + \frac{\overset{k+1}{p}_{i+1} - 2\overset{k+1}{p_i} + \overset{k+1}{p}_{i-1}}{h^2}\,,$$

or

$$A\overset{k+1}{p}_{i-1} - B\overset{k+1}{p_i} + C\overset{k+1}{p}_{i+1} = -\,P_i\,, \tag{11}$$

$$A_1\overset{k+1}{w}_{i-1} - B_1\overset{k+1}{w_i} + C_1\overset{k+1}{w}_{i+1} = -\,W_i - \gamma_1 R_i\,, \quad i = 1, 2, ..., I - 1\,, \tag{12}$$

where

$$A = A_1 = \gamma\,; \quad B = B_1 = 1 + 2\gamma\,; \quad C = C_1 = \gamma\,; \quad \gamma = \frac{\overset{m}{a}_1\tau}{h^2}\,; \quad \gamma_1 = \frac{\tau}{h^2}\,; \quad P_i = \overset{k}{p_i}\,; \quad W_i = \overset{k}{w_i}\,;$$

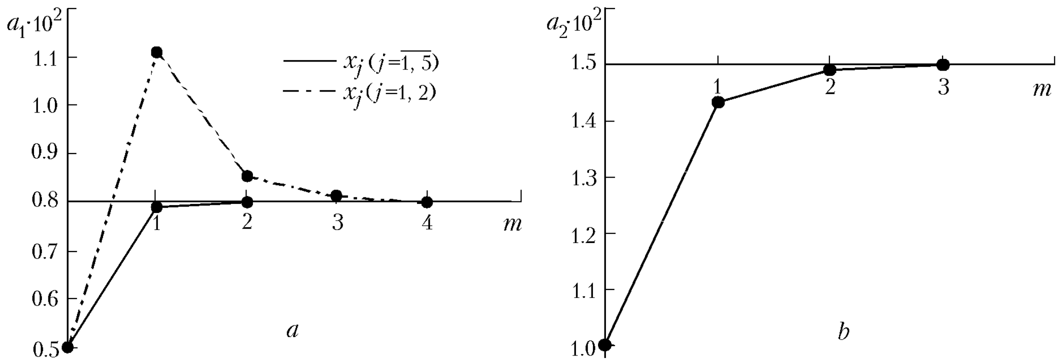

Fig. 1. Recovery of the values of a_1 and a_2 in the regime of reduction (a) and recovery (b) of pressure. a_1, a_2, m^2/sec.

$$R_i = p_{i+1}^{k+1} - 2p_i^{k+1} + p_{i-1}^{k+1} .$$

We approximate the initial and boundary conditions (8) and (9):

$$p_i^0 = p_0 , \quad p_0^{k+1} = p_{\text{w}} , \quad p_I^{k+1} = p_0 , \tag{13}$$

$$w_i^0 = p_0 , \quad w_0^{k+1} = 0 , \quad w_I^{k+1} = 0 . \tag{14}$$

To solve (11), (12) subject to (13), (14) we use the pivot method [12]. In the regime of pressure recovery, the algorithm of finding a_2 is similar to that used in finding a_1 in the regime of pressure reduction.

The grid splits the intercept [0; L] on the axis x into 300 intervals and the time portion [0; T] into 1000 ones. The "measurement data" were prepared at 5 × 10 "coordinate–time" points on the basis of the grid solution of the first equation of (1) with the given parameters p_0 = 50 MPa, p_{w} = 30 MPa, a_1 = 0.008 m^2/sec, L = 30 m, T = 1000 sec and of Eq. (2) with a_2 = 0.015 m^2/sec. The results of calculations on the identification of the parameters a_1 and a_2, when a_1^0 = 0.005 m^2/sec and a_2^0 = 0.01 m^2/sec are assigned as a zero approximation, are presented in Fig. 1. An analysis of the results shows that in the regime of pressure reduction the parameter a_1 comes to the equilibrium point on one side and is recovered practically with two iterations (Fig. 1a). When the initial information is prescribed at 2 × 10 "coordinate–time" points, the parameter approaches the equilibrium point with four iterations (Fig. 1a). In the regime of pressure recovery, when the initial information was prescribed at 5 × 10 "coordinate–time" points, the parameter a_2 approaches the equilibrium point from one side with three iterations (Fig. 1b). Thus, the reduction in the number of measurements along the coordinate which are used as the initial information in the identification problem leads to an increase in the number of iterations.

Let us now go to the determination of the parameters (1) by the method of deterministic moments. The principal advantage of using this method is that the relationships between the moments of the characteristics of the process and the model parameters are more simple than the relationship between the full solution and the model parameters of the process. Moreover, for some models it is impossible to obtain an analytical solution, while the moments can be determined [11] from relatively simple analytical expressions. The scheme of application of the method is as follows: first, one finds the solution of the hydrodynamic problem in the Laplace images. Then, the Laplace transformation properties are used, which allow one to write, in an analytical form, the dependence of the moments of solution of the direct problem on the parameters entering into Eq. (1). From these dependences the unknown parameters are then determined.

We will consider the first equation of (1) in the final bed [0, L] with initial and boundary conditions (8). There is also an additional condition

$$p\,(x_1, t) = \tilde{p}_1\,(t)\,, \quad x_1 \in [0, L]\,. \tag{15}$$

The second equation of (1) in the regime of pressure recovery will be considered under conditions (10) and additional condition

$$p\,(x_1, t) = \tilde{p}_2\,(t)\,. \tag{16}$$

Our task is to determine the parameters a_1 and a_2. Going over to the Laplace image

$$\bar{p}\,(x, s) = \int_0^\infty \exp\,(-\,st)\,p\,(x, t)\,dt\,,$$

from the first equation of (1) and from (8), (15) we obtain

$$\frac{d^2\bar{p}}{dx^2} - \frac{s}{a_1}\,\bar{p} = -\,\frac{p_0}{a_1}\,, \tag{17}$$

$$\bar{p}\,(0, s) = \frac{p_{\mathrm{w}}}{s}\,, \quad \bar{p}\,(L, s) = \frac{p_0}{s}\,, \tag{18}$$

$$\bar{p}\,(x_1, s) = \bar{\tilde{p}}_1\,(s)\,. \tag{19}$$

Substituting the solution of (17), (18)

$$\bar{p}\,(x, s) = \frac{(p_{\mathrm{w}} - p_0)\,\sinh\,[(L - x)\,k_1] + p_0\,\sinh\,(k_1 L)}{s\,\sinh (k_1 L)}$$

into Eq. (19) transformed as

$$\frac{p_{\mathrm{st}}}{s} + \Delta\bar{p}_1\,(s) = \bar{\tilde{p}}_1\,(s)\,,$$

where $\Delta\bar{p}_1(s) = \bar{p}(x_1, s) - \frac{p_{\mathrm{st}}}{s}$, we find

$$\frac{p_{\mathrm{st}}}{s} + \Delta\bar{p}_1\,(s) = \frac{(p_{\mathrm{w}} - p_0)\,\sinh\,[(L - x_1)\,k_1] + p_0\,\sinh\,(k_1 L)}{s\,\sinh\,(k_1 L)}\,, \quad k_1 = \sqrt{\frac{s}{a_1}}\,. \tag{20}$$

We will expand each term of (20) into a power series in s. Moreover, we will avail ourselves of the series expansion of the function $\sinh x$ and

$$\Delta\bar{p}_1\,(s) = \Delta p_{10} - s\Delta p_{11} + \frac{s^2}{2}\,\Delta p_{12} - \ldots\,, \quad \Delta p_{1l} = \int_0^\infty t^l \Delta p_1\,(t)\,dt\,, \quad l = 0, 1, \ldots\,.$$

As a result, we have

$$\left(L + \frac{L^3}{3!}\,k_1^2 + \ldots\right)\left(p_{\mathrm{st}} + s\Delta p_{10} - s^2\Delta p_{11} + \ldots\right) =$$

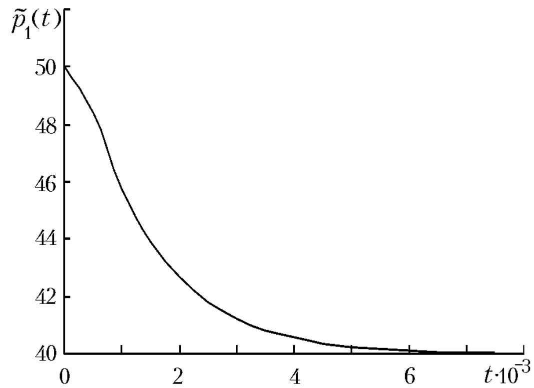

Fig. 2. Graph of the function $\tilde{p}_1(t)$. t, sec; $\tilde{p}_1(t)$, MPa.

$$=\left[(p_w-p_0)\,(L-x_1)+p_0L\right]+\left[\frac{(p_w-p_0)\,(L-x_1)^3}{3!}+\frac{p_0L^3}{3!}\right]k_1^2+\dots\,. \tag{21}$$

Equating the coefficients at 1, s, ... in (21), we obtain relationships for determining p_{st} and a_1. For this purpose it is sufficient to have two such relations:

$$p_{st}L=p_wL+(p_0-p_w)\,x_1\,,\quad 6La_1\Delta p_{10}=(p_w-p_0)\,(L-x_1)^3+(p_0-p_{st})\,L^3\,,$$

from which we determine

$$a_1=\frac{(p_w-p_0)\,(L-x_1)^3+(p_0-p_{st})\,L^3}{6La\Delta p_{10}}\,, \tag{22}$$

where $p_{st} = p_w+\dfrac{p_0-p_w}{L}x_1$, $\Delta p_{10} = \int\limits_0^\infty \Delta p_1(t)dt = \int\limits_0^\infty (\tilde{p}_1(t) - p_{st})dt$.

In Eq. (15) $\tilde{p}_1(t)$ is determined as the solution of the corresponding direct problem at a_1 = 0.008 m^2/sec as presented in Fig. 2. Calculations by Eq. (22) were performed at x_1 = 5 m, L = 10 m, p_0 = 50 MPa, p_w = 30 MPa, Δp_{10} = 15,631.8926 MPa·sec. The resulting calculated value a_1 = 0.007996 m^2/sec almost coincides with the given value a_1 = 0.008 m^2/sec.

Now, we will consider the second equation of (1) subject to (10), (16) and determine the parameter a_2. Having applied the Laplace transformation, we obtain

$$\frac{d^2\bar{p}}{dx^2}-\frac{s}{a_2}\bar{p}=\frac{p_1(x)}{a_2}\,, \tag{23}$$

$$\left.\frac{d\bar{p}}{dx}\right|_{x=0}=0\,,\quad \bar{p}\,(L,s)=\frac{p_0}{s}\,, \tag{24}$$

$$\bar{p}\,(x_1,s)=\bar{\tilde{p}}_2\,(s)\,. \tag{25}$$

The solution of (23) and (24) has the form

$$\bar{p}(x,s) = -\frac{1}{2k_3}\left[\exp(k_2 x)\int_0^x \exp(-k_2 u)\, p_1(u)\, du - \exp(-k_2 x)\int_0^x \exp(k_2 u)\, p_1(u)\, du\right] +$$

$$+\frac{\cosh(k_2 x)}{\cosh(k_2 L)}\left\{\frac{1}{2k_3}\left[\exp(k_2 L)\int_0^L \exp(-k_2 u)\, p_1(u)\, du -\right.\right.$$

$$\left.\left.-\exp(-k_2 L)\int_0^L \exp(k_2 u)\, p_1(u)\, du\right] + \frac{p_0}{s}\right\}, \quad k_2 = \sqrt{\frac{s}{a_2}}\,, \quad k_3 = \sqrt{a_2 s}\,.$$

At $x = x_1$, having substituted the following relations into the modified condition (25):

$$\frac{p_0}{s} - \Delta\bar{p}_2(s) = \bar{\tilde{p}}_2(s)\,, \quad \Delta p_2(t) = p_0 - p(x_1, t)\,,$$

we obtain

$$\frac{p_0}{s} - \Delta\bar{p}_2(s) = -\frac{1}{2k_3}\left[\exp(k_2 x_1)\int_0^{x_1} \exp(-k_2 u)\, p_1(u)\, du - \exp(-k_2 x_1)\,\times\right.$$

$$\left.\times\int_0^{x_1} \exp(k_2 u)\, p_1(u)\, du\right] + \frac{\cosh(k_2 x_1)}{\cosh(k_2 L)}\left\{\frac{1}{2k_3}\left[\exp(k_2 L)\int_0^L \exp(-k_2 u)\, p_1(u)\, du -\right.\right.$$

$$\left.\left.-\exp(-k_2 L)\int_0^L \exp(k_2 u)\, p_1(u)\, du\right] + \frac{p_0}{s}\right\}. \tag{26}$$

We will expand in a power series in s each term of Eq. (26) using the series expansion of the functions cosh x, exp (x) and

$$\Delta\bar{p}_2(s) = \Delta p_{20} - s\Delta p_{21} + \frac{s^2}{2}\Delta p_{22} - \ldots\,, \quad \Delta p_{2l} = \int_0^\infty t^l \Delta p_2(t)\, dt\,, \quad l = 0, 1, \ldots\,.$$

We have

$$a_2\left(1 + \frac{L^2}{2a_2}s + \ldots\right)\left(\frac{p_0}{s} + \Delta p_{20} - s\Delta p_{21} + \ldots\right) = -\left(s + \frac{L^2}{2a_2}s^2 + \ldots\right)\times$$

$$\times\left[x_1\int_0^{x_1} p_1(u)\, du - \int_0^{x_1} u p_1(u)\, du + \ldots\right] + \left(s + \frac{x_1^2}{2a_2}s^2 + \ldots\right)\times$$

$$\times\left[L\int_0^L p_1(u)\, du - \int_0^L u p_1(u)\, du + \ldots\right] + a_2\left(1 + \frac{x_1^2}{2a_2} + \ldots\right)p_0\,. \tag{27}$$

Equating the coefficients 1, s, ... in (27), we obtain relations from which we find a_2:

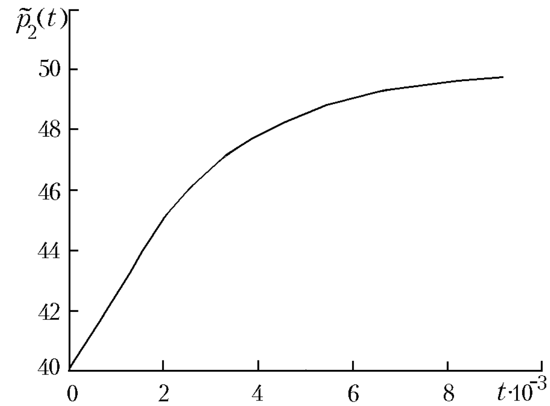

Fig. 3. Graph of the function $\tilde{p}_2(t)$. t, sec; $\tilde{p}_2(t)$, MPa.

$$a_2 = \frac{1}{\Delta p_{20}} \left[\frac{L^2 - x_1^2}{2} p_0 + \int_0^{x_1} (x_1 - u)\, p_1(u)\, du + \int_0^{L} (u - L)\, p_1(u)\, du \right], \tag{28}$$

where

$$\Delta p_{20} = \int_0^{\infty} \Delta p_2(t)\, dt = \int_0^{\infty} (p_0 - \tilde{p}_2(t))\, dt\,.$$

To perform calculations by (28) we first solve numerically the second equation of (1) subject to (10) at the given a_2 = 0.015 m^2/sec, then we calculate $\tilde{p}_2(t)$. The graph of the function $\tilde{p}_2(t)$ is shown in Fig. 3. Calculations by Eq. (28) at x_1 = 5 m, L = 10 m, p_0 = 50 MPa, p_w = 30 MPa, Δp_{20} = 2171.0612 MPa·sec yield the value a_2 = 0.017 m^2/sec close to that given.

Thus, the method of deterministic moments allows one to find simple formulas for determining the unknown parameters of the problem, in the given case — the coefficients of the equation. The calculations show that expressions (22) for a_1 and (28) for a_2 are stable against perturbations of the functions $\tilde{p}_1(t)$ and $\tilde{p}_2(t)$, respectively. However, the method of deterministic moments is applicable only to linear problems, which limits its use in a wide class of important, interesting problems that involve nonlinearity. Due to its broad application, the identification method is preferred to the method of deterministic moments.

NOTATION

a_1 and a_2, coefficients of piesoconductivity of a bed in the regime of reduction and recovery of pressure, m^2/sec; $\overset{m}{a}_1$ and $\overset{m}{a}_2$, mth approximations of a_1 and a_2, m^2/sec; h, grid step in the x direction, m; $J(a_1)$, minimized functional; L, bed length, m; m, number of iteration; p, current pressure, MPa; p_0, initial pressure in a bed, MPa; $p_1(x)$, pressure distribution at the end of the reduction phase, MPa; p_w, pressure in a well, MPa; p_i^k, w_i^k, grid solutions corresponding to the point (x_i, t_k); p_{st}, stationary solution of a direct problem at the point $x = x_1$, MPa; $\bar{p}$, Laplace transformation of p; $\tilde{p}_1(t)$ and $\tilde{p}_2(t)$, functions determining the values of p at the point $x = x_1$ in the regime of reduction and recovery of pressure, MPa; s, parameter of Laplace transformation; t, time, sec; T, maximum time of pressure reduction, sec; u, variable of integration over the coordinate x, m; x, linear coordinate, m; $z_j(t)$ $(j = \overline{1, n})$, function of a change of p at x_j $(j = \overline{1, n})$, i.e., $p(x_j, t) = z_j(t)$ $j = \overline{1, n}$, MPa; ε_1, ε_2, ε_3, given small quantities; τ, grid step in time, sec. Subscripts and superscripts: st, steady-state regime; w, well; ↓↑, correspond to the processes of reduction and recovery of pressure; m above functions denotes that they were calculated at $a_1 = \overset{m}{a}_1$ (or $a_2 = \overset{m}{a}_2$); overbar corresponds to Laplace image.

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